

SUPPLEMENTARY MATERIALS  
Some Tasks Demands Induce Collapsing Bounds:  
Evidence from a Behavioral Analysis

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**Abstract**

Traditional models of choice response time assume that sensory evidence accumulates for choice alternatives until a threshold amount of evidence has been obtained. Although some researchers have characterized the threshold as varying randomly from trial to trial, these investigations have all assumed that the threshold remains fixed across time within a trial. Despite decades of successful applications of these models to a variety of experimental manipulations, the time-invariance assumption has recently been called into question, and a time-variant alternative implementing collapsing decision thresholds has been proposed instead. Here, we investigated the fidelity of the collapsing threshold assumption by assessing relative model fit to data from a highly constrained experimental design that coupled a within-subject mixture of two classic response time paradigms – interrogation and free response – within a random dot motion (RDM) task. Overall, we identified strong evidence in

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favor of collapsing decision thresholds, suggesting that subjects may adopt a dynamic decision policy due to task characteristics, specifically to account for the mixture of response time paradigms and motion strengths across trials in the mixed response signal task. We conclude that time-variant mechanisms may serve as a viable explanation for the strategy used by human subjects in our task.

*Keywords:* fixed boundaries, collapsing boundaries, likelihood-free methods, random dot motion, interrogation

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1 Here, we present additional details, tables, figures, and analyses not in-  
2 cluded in the main text. In the first section, we provide details about the  
3 evolution of evidence in the diffusion decision model (DDM; Ratcliff, 1978;  
4 Ratcliff and Rouder, 1998). Second, we present the a table that displays the  
5 arrangement of parameters featured in each of the 16 model variants under  
6 consideration in our comparison. Next, we present the unstandardized (i.e.,  
7 raw) model fit statistics. In the fourth section, we present an additional  
8 model comparison analysis using approximated posterior model probabilities  
9 and an assessment of model fit to observed data using bar plots. The fifth  
10 section provides the numerical approximated Bayes factor scores for each  
11 subject. The sixth section provides additional analyses exploring the im-  
12 pact of practice on model fits. The final section provides the inferred task  
13 representation plots for the remaining 12 subjects not included in the main  
14 text.

## 15 **1. The Diffusion Decision Model**

16 The diffusion decision model (DDM; Ratcliff, 1978; Ratcliff and Rouder,  
17 1998) assumes that, on the presentation of a stimulus  $S$  (e.g., a random  
18 dot display), an observer accumulates sensory evidence as noisy samples and  
19 integrates these samples over the course of a trial. The integration process  
20 can be described in terms of the differential equation

$$de(t) = vdt + sdW, \tag{1}$$

21 where  $e(t)$  is the value of the integrated evidence at time  $t$ ,  $v$  is the mean of  
22 the accumulated samples (i.e., drift rate), and  $s$  represents the moment-to-  
23 moment variability in the drift rate (Ratcliff, 1978; Ratcliff and Rouder, 1998;

24 Turner et al., 2017). The parameter  $v$  dictates the strength of evidence, so in  
 25 the dot motion task,  $s$  would indicate the strength of coherence toward one  
 26 direction or another. However, due to some perceptual biases, the motion  
 27 coherence may not map directly onto the diffusion process in Equation 1.  
 28 Denoting the coherence variable as  $C$ , and  $\zeta$  as the coherence scaling factor,

$$v = C\zeta .$$

29 The values of  $C$  can be positive, arbitrarily indicating strength of motion to  
 30 the right, or negative, indicating motion to the left. If we assume that no  
 31 decision bound interferes with the integration process (e.g., in the interroga-  
 32 tion paradigm), Equation 1 dictates how observers integrate evidence from  
 33 the initial presentation of the stimulus until the time it is removed from the  
 34 screen. If  $W$  in Equation 1 denotes a Wiener process, as it does in the DDM  
 35 (see Stone, 1960; Smith, 2000, for a detailed overview), the sensory evidence  
 36  $e(t)$  is assumed to be sampled continuously over time from a normal distri-  
 37 bution such that  $e(t) \sim N(\mu(t), \sigma(t))$  with mean  $\mu(t)$  standard deviation  
 38  $\sigma(t)$ , such that

$$\begin{aligned} \mu(t) &= vt \\ \sigma(t) &= s\sqrt{t}. \end{aligned} \tag{2}$$

39 When modeling data from an interrogation paradigm, the typical approach is  
 40 to compare the state of evidence at the interrogation time  $t$  to some criterion  
 41  $c(t)$ . Given our choices about how positive evidence relates to rightward  
 42 motion, a rightward response would be generated if  $e(t) > c(t)$ . Otherwise,  
 43 a leftward response would be made.

44 The DDM includes several sources of variability beyond simple sensory  
 45 integration that complicate Equation 1. These extra sources of variability  
 46 have proven effective in accounting for a number of empirical benchmarks  
 47 in decision-making tasks (Ratcliff and McKoon, 2008; Ratcliff and Rouder,  
 48 1998; Ratcliff and Tuerlinckx, 2002). The first source of variability is the  
 49 within-trial variability term  $\sigma_w$  already discussed in Equation 1. The sec-  
 50 ond source of variability is trial-to-trial (i.e., between-trial) variability in the  
 51 drift rate, which results from variability in the mean stimulus value of the  
 52 stimuli used in the task. In line with Equation 1, this variability is imple-  
 53 mented by perturbing the trial-specific value of  $\mu$ , typically from a normal  
 54 distribution such that  $\mu \sim \mathcal{N}(v, \eta)$ . Here, the standard deviation parameter  
 55  $\eta$  represents the between-trial variability in drift rate. The third source of  
 56 variability is variability in the starting point of the diffusion process. This  
 57 source is typically thought of as being independent of the sensory proper-  
 58 ties of the stimulus itself, and instead is internal to the observer. Starting  
 59 point variability is implemented in a similar manner as the between-trial  
 60 variability in drift: on each trial, the initial evidence value is perturbed by  
 61 a normal distribution centered at  $z_0$  with a standard deviation  $S_0$ , such that  
 62  $e(t = 0) \sim \mathcal{N}(z_0, S_0)$ . Between-trial variability in the starting point allowed  
 63 the diffusion model to account for the relative speeds of errors and correct  
 64 responses in the SAT manipulation (e.g., Ratcliff et al., 2006; Starns and Rat-  
 65 cliff, 2010; White et al., 2009). Considering these three sources of variability,  
 66 the mean  $\mu(t)$  and standard deviation  $\sigma(t)$  from Equation 2 becomes

$$\begin{aligned}
 \mu(t) &= vt + z_0 \\
 \sigma(t) &= \sqrt{S_0^2 + s^2t + \eta^2t^2}.
 \end{aligned}
 \tag{3}$$

67 The relative importance of each source of variability is investigated in the  
68 main text.

## 69 **2. Model Variants**

70 The arrangement of parameters featured in each of the 16 variants is dis-  
71 played in Table 1. Model variants were determined by the type of boundary  
72 (time-invariant versus time-variant) and the combination of freely estimated  
73 and fixed parameters. If a parameter is freely estimated in the model, it is  
74 denoted as “FE” in the table. If the parameter is fixed, it is denoted as “F.”  
75 If a parameter is absent from the model (e.g, the parameters associated with  
76 the collapsing boundary that are not included in the time-variant models),  
77 the space is left blank. This model order (1-16) will remain consistent in  
78 the following sections, and models will be referred to by their model number  
79 denoted in Table 1.

## 80 **3. Raw Model Fit Statistics**

81 Tables 1 and 2 provide the model fit statistics on their original scale  
82 for each of the 16 models and each subject. The best fitting model for each  
83 subject is in boldface type. For all subjects, the best fitting model is a variant  
84 of the time-variant diffusion model with collapsing boundaries, with Class 3  
85 ( $\lambda$  free) accounting for the most subjects overall. The second best class of  
86 models was Class 4, which freely estimated both  $a'$  and  $\lambda$ , best accounting  
87 for 5 out of 14 subjects. The remaining two subjects were best accounted  
88 for by either Class 1 (i.e., the time-invariant class) or Class 2, which freely  
89 estimated  $a'$ .

Table 1: **Model Variants.** Each row corresponds to a specific model we fit to data, where parameters (i.e., columns) were either freely estimated (FE) or fixed (F). If a parameter is not applicable, such as in the time-invariant models, the space is left blank.

| Model | Decision Boundaries | $a$ | $v$ | $z_0$ | $s$ | $\tau_{er}$ | $S_\tau$ | $S_0$ | $\eta$ | $a'$ | $\lambda$ | $k$ | Free Parameters |
|-------|---------------------|-----|-----|-------|-----|-------------|----------|-------|--------|------|-----------|-----|-----------------|
| 1     | Fixed               | FE  | FE  | FE    | F   | FE          | FE       | F     | F      |      |           |     | 5               |
| 2     | Fixed               | FE  | FE  | FE    | F   | FE          | FE       | F     | FE     |      |           |     | 6               |
| 3     | Fixed               | FE  | FE  | FE    | F   | FE          | FE       | FE    | F      |      |           |     | 6               |
| 4     | Fixed               | FE  | FE  | FE    | F   | FE          | FE       | FE    | FE     |      |           |     | 7               |
| 5     | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | F     | F      | FE   | F         | F   | 6               |
| 6     | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | F     | FE     | FE   | F         | F   | 7               |
| 7     | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | FE    | F      | FE   | F         | F   | 7               |
| 8     | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | FE    | FE     | FE   | F         | F   | 8               |
| 9     | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | F     | F      | F    | FE        | F   | 6               |
| 10    | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | F     | FE     | F    | FE        | F   | 7               |
| 11    | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | FE    | F      | F    | FE        | F   | 7               |
| 12    | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | FE    | FE     | F    | FE        | F   | 8               |
| 13    | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | F     | F      | FE   | FE        | F   | 7               |
| 14    | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | F     | FE     | FE   | FE        | F   | 8               |
| 15    | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | FE    | F      | FE   | FE        | F   | 8               |
| 16    | Collapsing          | FE  | FE  | FE    | F   | FE          | FE       | FE    | FE     | FE   | FE        | F   | 9               |

Table 2: **Model fit statistics for Subjects 1-7.** boldface type represents the lowest BIC value for each subject.

| Model | 1             | 2             | 3             | 4             | 5             | 6             | 7             |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1     | 1049.27       | 1042.37       | 1093.19       | 1008.76       | 803.54        | 919.73        | 982.01        |
| 2     | 953.62        | 928.95        | 1134.93       | 863.97        | 784.63        | 807.44        | 986.10        |
| 3     | 885.27        | 932.26        | 781.04        | 824.48        | 801.99        | 839.36        | 910.01        |
| 4     | 1001.95       | 908.22        | 1046.31       | 891.85        | 775.69        | 791.2         | 945.51        |
| 5     | 926.87        | 920.86        | 991.25        | 874.36        | 798.97        | 867.23        | 913.18        |
| 6     | 889.56        | 1134.45       | 1040.52       | 761.77        | 762.4         | 732.06        | 832.11        |
| 7     | 833.55        | 930.09        | 780.56        | 821.79        | 767.78        | 833.68        | 892.91        |
| 8     | 890.09        | 925.9         | 786.16        | 894.18        | 781.46        | 844.88        | 912.16        |
| 9     | 1028.06       | 1028.89       | 882.41        | 786.15        | 772.47        | 775.89        | 739.83        |
| 10    | 783.61        | 902.80        | 1041.80       | <b>712.44</b> | 772.06        | <b>668.09</b> | 729.12        |
| 11    | 769.1         | 889.06        | 772.27        | 791.82        | 779.11        | 754.18        | <b>715.78</b> |
| 12    | <b>763.61</b> | 972.59        | 841.95        | 822.66        | 793.02        | 805.73        | 793.89        |
| 13    | 773.29        | 1055.65       | 930.19        | 842.7         | 780.21        | 808.74        | 971.92        |
| 14    | 788.95        | <b>864.94</b> | 900.79        | 731.19        | <b>759.90</b> | 725.36        | 806.56        |
| 15    | 784.72        | 922.83        | <b>769.63</b> | 805.99        | 789.51        | 783.16        | 827.23        |
| 16    | 801.43        | 970.06        | 808.69        | 849.1         | 788.87        | 850.85        | 801.99        |



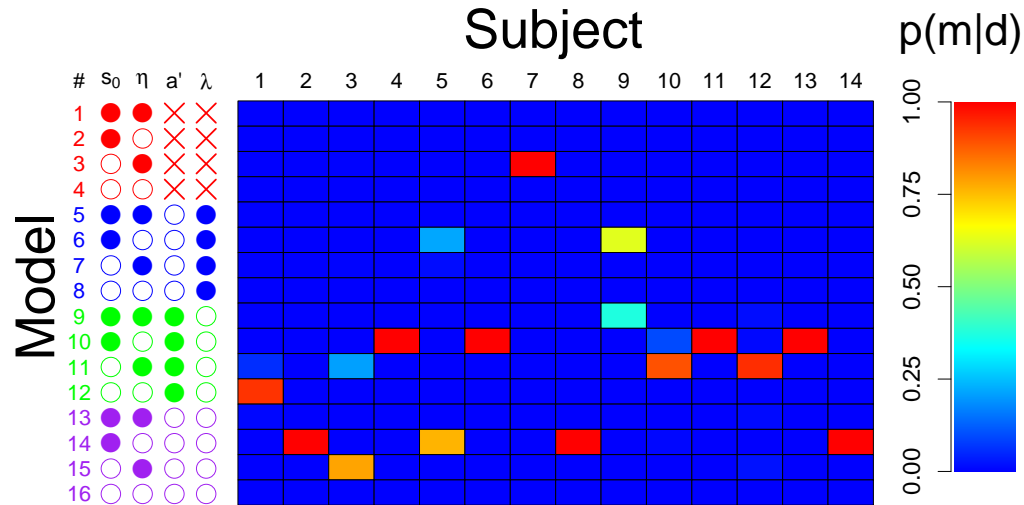
Table 3: **Model fit statistics for Subjects 8-14.** boldface type represents the lowest BIC value for each subject.

| Model | 8             | 9             | 10            | 11            | 12            | 13            | 14            |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1     | 775.71        | 991.92        | 724.68        | 992.44        | 501.22        | 915.63        | 769.59        |
| 2     | 634.18        | 883.91        | 687.46        | 989.68        | 529.26        | 814.55        | 680.70        |
| 3     | 767.44        | 959.03        | 706.02        | 947.96        | 499.26        | 730.39        | 758.59        |
| 4     | 616.46        | 916.72        | 696.79        | 916.11        | 503.74        | 720.85        | 705.31        |
| 5     | 652.17        | 938.47        | 687.78        | 970.18        | 491.42        | 777.46        | 757.84        |
| 6     | 591.90        | <b>844.78</b> | 674.59        | 857.79        | 496.09        | 730.25        | 655.59        |
| 7     | 658.51        | 919.95        | 677.78        | 937.88        | 496.51        | 700.61        | 734.95        |
| 8     | 627.44        | 908.64        | 672.07        | 829.66        | 497.42        | 774.14        | 718.03        |
| 9     | 607.86        | 845.82        | 660.56        | 817.11        | 484.81        | 700.03        | 676.48        |
| 10    | 631.55        | 893.08        | 652.55        | <b>702.74</b> | 485.65        | <b>671.05</b> | 677.13        |
| 11    | 607.87        | 877.35        | <b>648.10</b> | 741.94        | <b>476.54</b> | 718.87        | 692.81        |
| 12    | 637.92        | 891.72        | 666.32        | 759.94        | 501.03        | 775.38        | 639.13        |
| 13    | 638.97        | 875.00        | 694.13        | 846.25        | 485.23        | 907.59        | 691.22        |
| 14    | <b>546.29</b> | 968.94        | 657.48        | 743.69        | 532.9         | 684.89        | <b>610.12</b> |
| 15    | 629.47        | 878.16        | 666.24        | 791.53        | 484.00        | 714.01        | 718.07        |
| 16    | 570.64        | 870.55        | 677.58        | 772.04        | 495.96        | 705.65        | 678.64        |

## 90 4. Model Comparison

91 In addition to comparing subjects using standardized BIC scores, we also  
92 compared subjects by calculating the approximate posterior model probabil-  
93 ities as given by Wasserman (2000). Figure 1 shows the resulting posterior  
94 model probability values obtained. Each model performance statistic is color  
95 coded according to the legend on the right hand side. For the approximate  
96 posterior model probability, values closer to one (warmer colors) indicate the  
97 better fit, relative to the 15 other model variants for each subject. The left  
98 panel of Figure 1 provides a schematic of the various models investigated  
99 here. Parameters are represented as nodes in each column, where param-  
100 eters that were freely estimated are empty, parameters that were fixed are  
101 solid, and parameters that are not applicable are shown as “x”s. From left to  
102 right, the columns are the model numbers, between-trial variability in start-  
103 ing point  $S_0$ , between-trial variability in drift  $\eta$ , the asymptotic boundary  
104 setting  $a'$ , and the scaling parameter of the collapsing decision boundary  $\lambda$ .  
105 For ease of discussion, the models were grouped into four classes: Models  
106 1-4 (Class 1; red) are the time-invariant models, Models 5-8 (Class 2; blue)  
107 are the time-variant models with  $a'$  free and  $\lambda$  fixed, Models 9-12 (Class 3;  
108 green) are the time-variant models with  $a'$  fixed and  $\lambda$  free, and Models 13-16  
109 (Class 4; purple) are the time-variant models with both  $a'$  and  $\lambda$  free.

110 Figure 1 shows that all subject’s data are best explained by one or two  
111 model variants, and these model variants are some variant of the time-  
112 invariant diffusion model. Across subjects, most of the models with high  
113 posterior probabilities belong to Class 3 ( $\lambda$  is free), but Class 4 also performs  
114 well. These results suggest that freeing  $\lambda$  (Class 3) substantially improves



115 model performance for the majority of subjects in our study. However, free-  
116 ing  $a'$  in addition to  $\lambda$  (Class 4) improved model fit in such a way that  
117 the penalty induced by the additional parameter was not enough to damage  
118 the performance of the model. Again, while the posterior model probability  
119 lacks the finer gradient of the standardized BIC model comparison, the clear  
120 worst performer is Class 1, with all four models showing 0% probability for  
121 all subjects.

#### 122 *4.1. Fit to Observed Data*

123 Figures 2, 3, 4, and 5 display the model fits to observed data in another  
124 way. Instead of plotting each model's fit against the observed data as in  
125 Figure 5 of the main text, Figures 2, 3, 4, and 5 show the accuracy (y-  
126 axis) for both the data (grey) and the model predictions (colors) as a bar  
127 plots for each interrogation time (x-axis) and for each level of coherency  
128 (columns). These figures allow us to see in greater detail how closely the  
129 model predictions are to the observed data and where the models have over-  
130 or under-predicted the accuracy relative to the observed data.

131 While the predictions across all four models are highly variable, Model  
132 4 and Model 14 tend to predict a different pattern of accuracy for the 0.15,  
133 0.25, and 0.35 coherency conditions. Whereas Model 7 and Model 11  
134 predict an increase in accuracy with an increase in interrogation time in these  
135 conditions, the accuracy as predicted by Models 4 and 14 tends to plateau  
136 for these conditions in the left panel (i.e., "All Responses") and decreases in  
137 the bottom right panel (i.e., "Responses After the Cue").

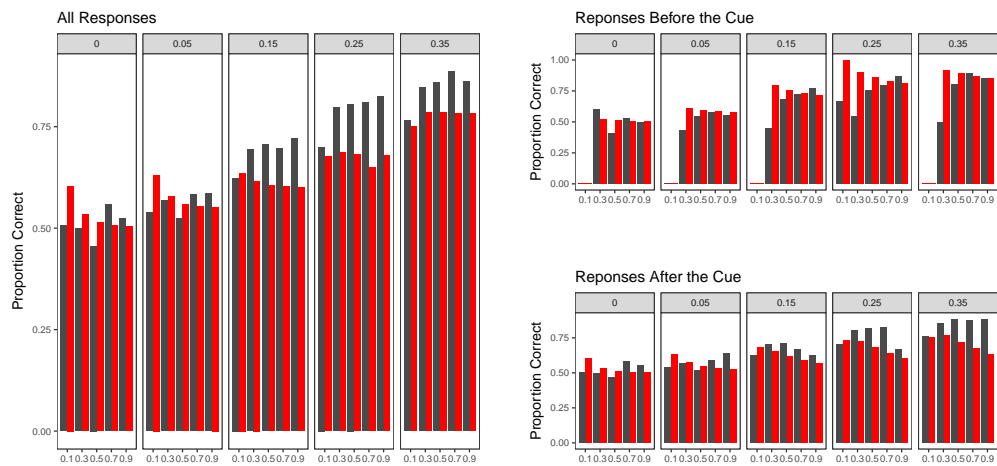


Figure 2: **Model Predictions of Accuracy Across Independent Variables versus Observed Data.** The left panel shows the observed accuracy (gray) and predicted accuracy from Model 4 (red) (*y*-axis) as bar plots for each interrogation time (*x*-axis) and for each coherency (panels), collapsed across response times. The right panel shows the same information as in the left panel, but separated based on whether the response was made prior to (top panel) or after (bottom panel) the stimulus disappeared.

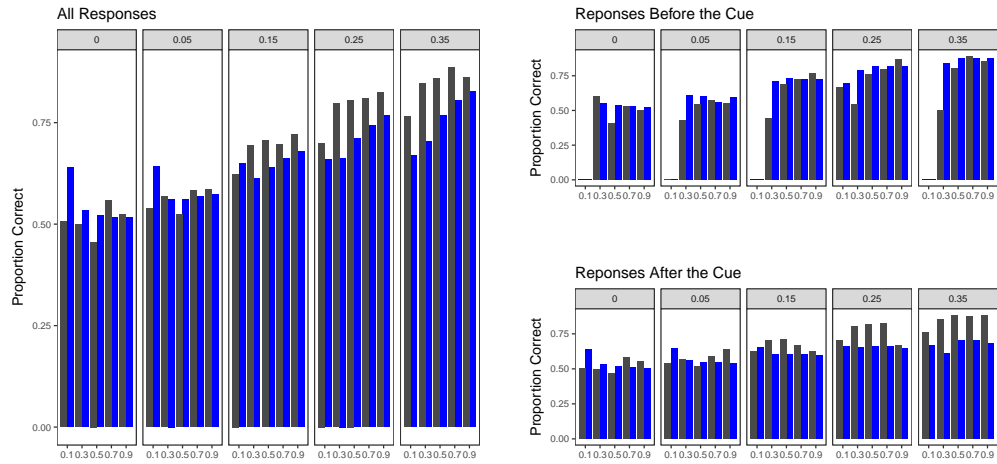


Figure 3: **Model Predictions of Accuracy Across Independent Variables versus Observed Data.** The left panel shows the observed accuracy (gray) and predicted accuracy from Model 7 (blue) ( $y$ -axis) as bar plots for each interrogation time ( $x$ -axis) and for each coherency (panels), collapsed across response times. The right panel shows the same information as in the left panel, but separated based on whether the response was made prior to (top panel) or after (bottom panel) the stimulus disappeared.

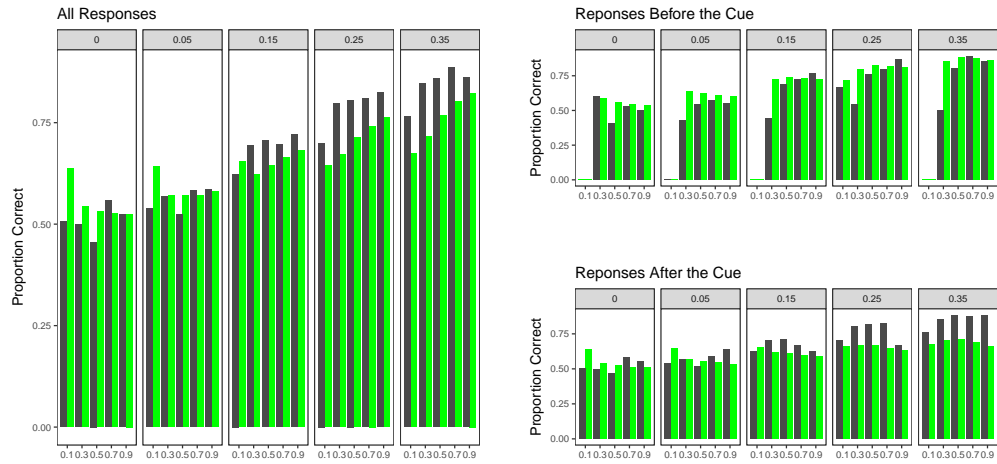


Figure 4: **Model Predictions of Accuracy Across Independent Variables versus Observed Data.** The left panel shows the observed accuracy (gray) and predicted accuracy from Model 11 (green) ( $y$ -axis) as bar plots for each interrogation time ( $x$ -axis) and for each coherency (panels), collapsed across response times. The right panel shows the same information as in the left panel, but separated based on whether the response was made prior to (top panel) or after (bottom panel) the stimulus disappeared.

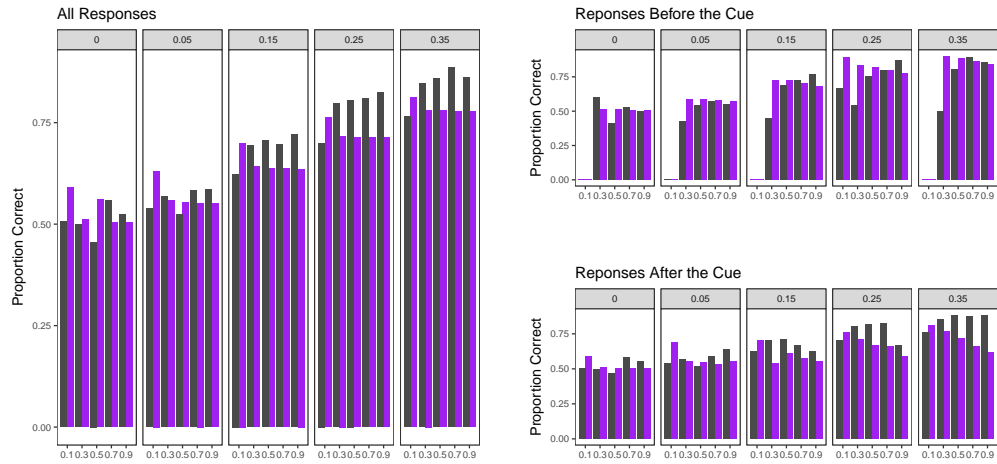


Figure 5: **Model Predictions of Accuracy Across Independent Variables versus Observed Data.** The left panel shows the observed accuracy (gray) and predicted accuracy from Model 14 (purple) ( $y$ -axis) as bar plots for each interrogation time ( $x$ -axis) and for each coherency (panels), collapsed across response times. The right panel shows the same information as in the left panel, but separated based on whether the response was made prior to (top panel) or after (bottom panel) the stimulus disappeared.



## 138 5. Bayes Factor Approximation

139 Table 3 displays the numerical approximated Bayes factor scores for each  
140 subject. Again, 13 of 14 subjects showed a preference for the time-invariant  
141 model, with the majority showing “strong” evidence for the time-variant  
142 model.

## 143 6. Practice

### 144 6.1. Practice Analysis

145 Figure 6 illustrates the differences in fit between a linear function and a  
146 power function for the mean (left panel) and the standard deviation (right  
147 panel) of response times across each of the 13 experimental blocks. Due to  
148 the large difference in response times between the first and second block, both  
149 the mean and standard deviations appear to be best fit by a power function  
150 ( $BIC_{mean} = 132.70$ ;  $BIC_{std} = 122.33$ ) versus a linear function ( $BIC_{mean} =$   
151  $142.66$ ;  $BIC_{std} = 131.10$ ).

### 152 6.2. Model Comparison with Trimmed Data

153 Given the presence of practice effects in our data and the potential that  
154 “split” in the split half analysis performed in Section 5.4 of the main text  
155 is an arbitrary distinction, we refit Models 4, 7, 11 and 14 (i.e., the best  
156 fitting model from each class) to each subject’s data after removing the first  
157 four blocks (i.e., 200 trials), which appeared to have the largest influence of  
158 practice. This model fit would allow us to see if removing the influence of  
159 practice impacted each model’s ability to account for the data. Due to the

Table 4: **Bayes factor comparison.** Approximated Bayes factor values after comparing the best fitting time-variant model (iModel 11, which includes between-trial variability in starting point and freely estimates  $\lambda$ ) to the best fitting time-invariant model (i.e., Model 4, where both sources of trial-to-trial variability are included) for each of the subjects in our data.

| Subject | Log(Bayes Factor) | Strength of Evidence | Model          |
|---------|-------------------|----------------------|----------------|
| 1       | 116.43            | Strong               | Time-variant   |
| 2       | 9.58              | Strong               | Time-variant   |
| 3       | 137.02            | Strong               | Time-variant   |
| 4       | 50.02             | Strong               | Time-variant   |
| 5       | -1.71             | Strong               | Time-invariant |
| 6       | 18.51             | Strong               | Time-variant   |
| 7       | 114.87            | Strong               | Time-variant   |
| 8       | 4.3               | Strong               | Time-variant   |
| 9       | 19.69             | Strong               | Time-variant   |
| 10      | 24.35             | Strong               | Time-variant   |
| 11      | 57.09             | Strong               | Time-variant   |
| 12      | 13.6              | Strong               | Time-variant   |
| 13      | 0.99              | Moderate             | Time-variant   |
| 14      | 6.25              | Strong               | Time-variant   |

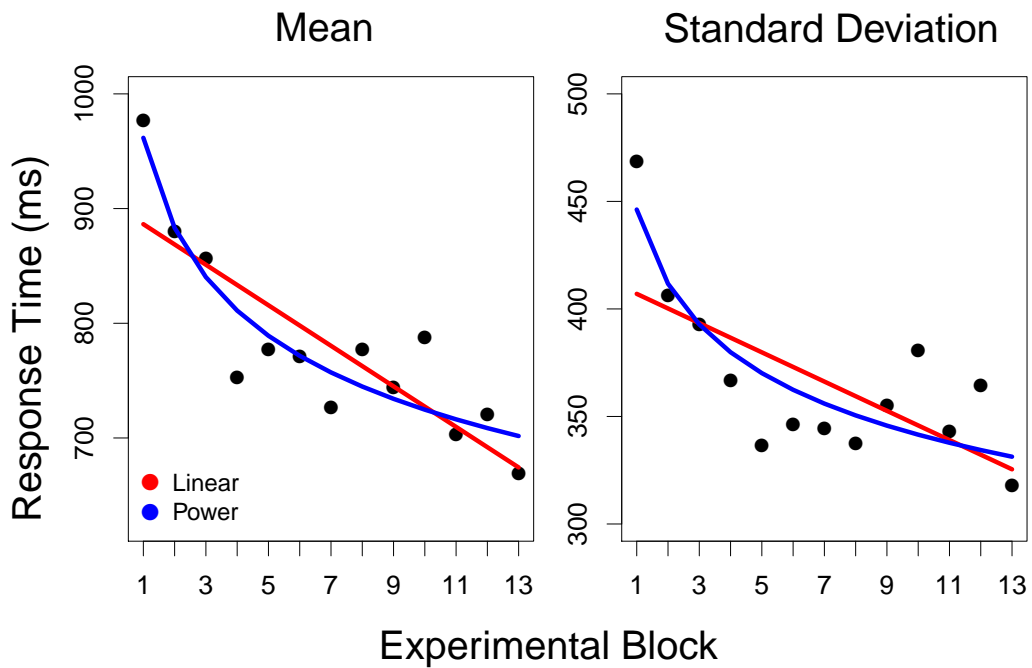


Figure 6: **Analysis of Practice Effects.** The left panel shows the best fitting linear (red) and power (blue) function to the mean response time across blocks. Similarly, the right panel shows the best fitting linear (red) and power (blue) function to the standard deviation of the response times across blocks.

160 removal of these four blocks, Subject 12 was dropped from the analysis due  
161 as only 9% of their data were remaining.

162 The results of these fits are in Figure 7, which shows the zBIC values,  
163 color coded according to the legend on the right side. Cooler colors represent  
164 lower BIC values and better model performance. While these fits are highly  
165 variable, the clear best fitting model across subjects is Model 11, with Models  
166 7, 14, and 4 following in that order. Overall, the majority of subjects are  
167 best fit by collapsing bound models, suggesting that time-variant mechanisms  
168 serve as a more viable explanation for how subjects complete the task even  
169 after removing the influence of practice.

## 170 **7. Inferred Task Representations**

171 Inferred task representation plots for the remaining 12 subjects are dis-  
172 played below. In these plots, the best fitting model is indicated by the color  
173 of the response time distribution in the top row of each figure. If the distribu-  
174 tion is red, then the subject's data are better fit by the time-invariant model  
175 (Model 4). If the distribution is blue, then the subject's data are better fit  
176 by the time-variant model (Model 16).

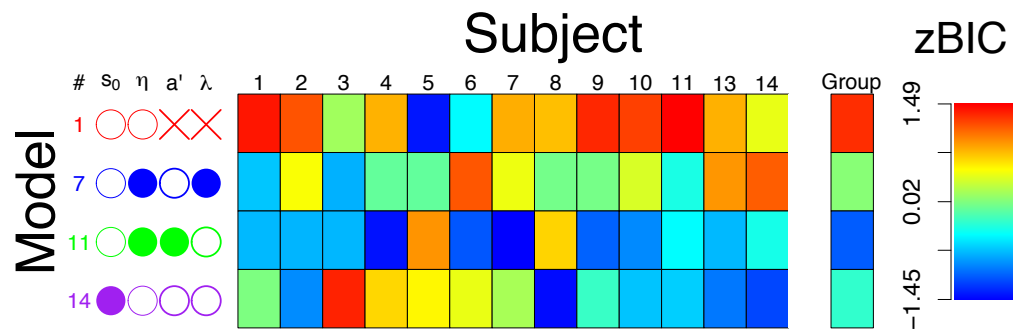


Figure 7: **Model Comparison of Trimmed Data.** Each box illustrates the z-scored BIC value obtained for each subject (columns) and model (rows) combination, color coded according to the legend on the right-hand side. Lower BIC values (i.e., cooler colors) correspond to better model performance. The panel on the left summarizes the models investigated here, where the columns correspond to the model number and the specific model parameters that were either fixed (filled circle), free to vary (empty circle) or not applicable (an “x” symbol). The models are color coded according to class: time-invariant (Model 4; red); time-variant with  $a'$  free (Model 7 ; blue); time-variant with  $\lambda$  free (Model 11; green); and time-variant with both  $a'$  and  $\lambda$  free (Models 13-16; purple).

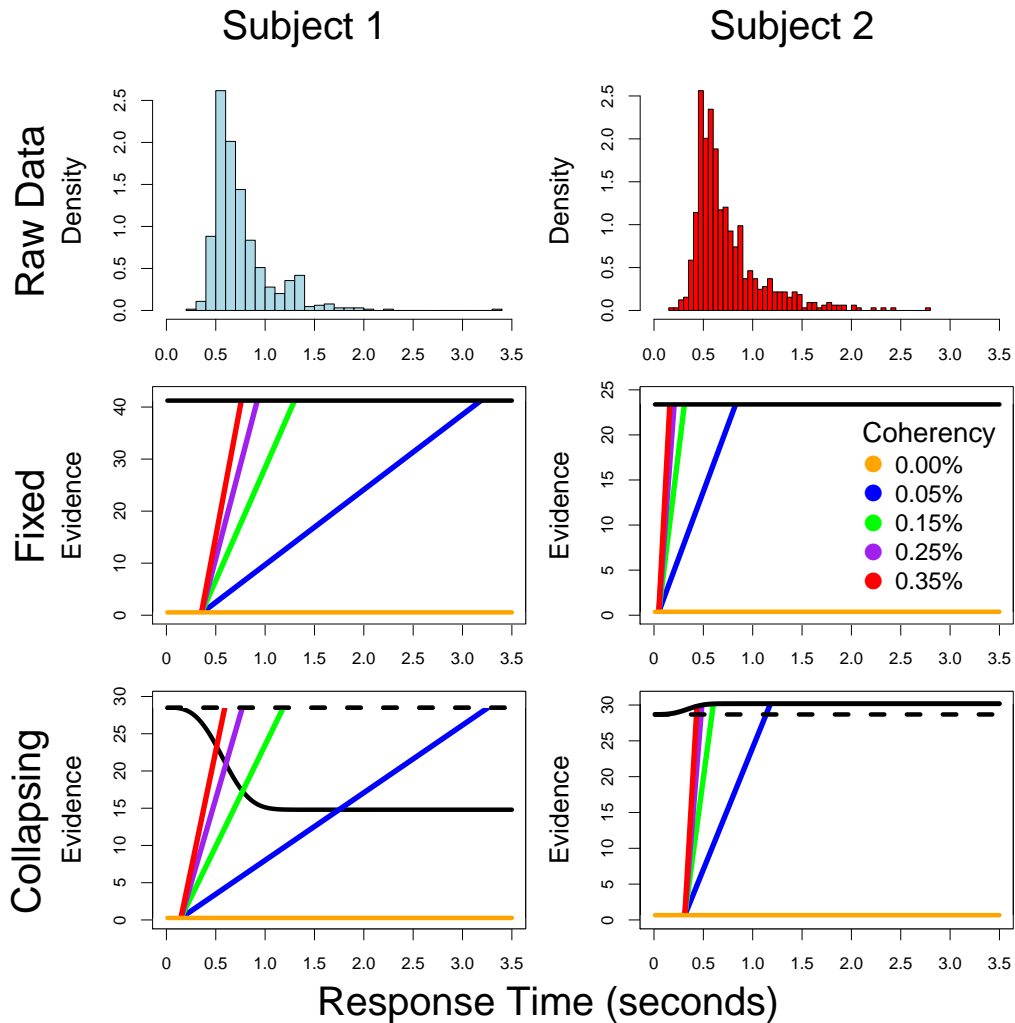


Figure 8: **Inferred Task Representations for subjects 1 and 2.** The columns correspond to two subjects' data. The top row shows the empirical response time distribution collapsed across choice and stimulus conditions. If the response time distribution is red, the subjects' data was best fit by the time-invariant model (Model 4). If the response time distribution is light blue, the subjects' data was best fit by the time-variant model (Model 16). The middle row shows the task representation inferred by a fixed boundary model, whereas the bottom row shows the task representation inferred by a collapsing boundary model. The drift rates for each coherence condition are shown, color coded according to the key in the middle right panel.

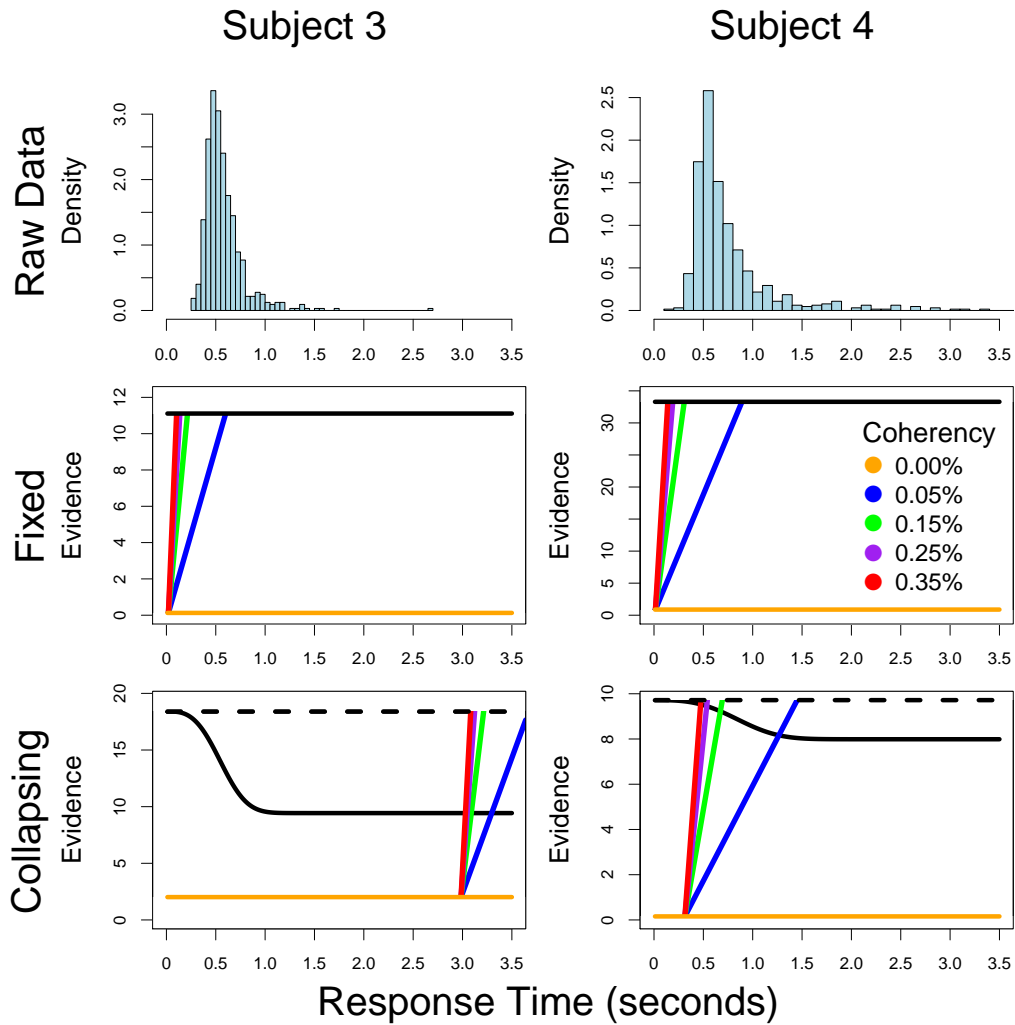


Figure 9: **Inferred Task Representations for Subjects 3 and 4.** The columns correspond to two subjects' data. The top row shows the empirical response time distribution collapsed across choice and stimulus conditions. If the response time distribution is red, the subjects' data was best fit by the time-invariant model (Model 4). If the response time distribution is light blue, the subjects' data was best fit by the time-variant model (Model 16). The middle row shows the task representation inferred by a fixed boundary model, whereas the bottom row shows the task representation inferred by a collapsing boundary model. The drift rates for each coherency condition are shown, color coded according to the key in the middle right panel.

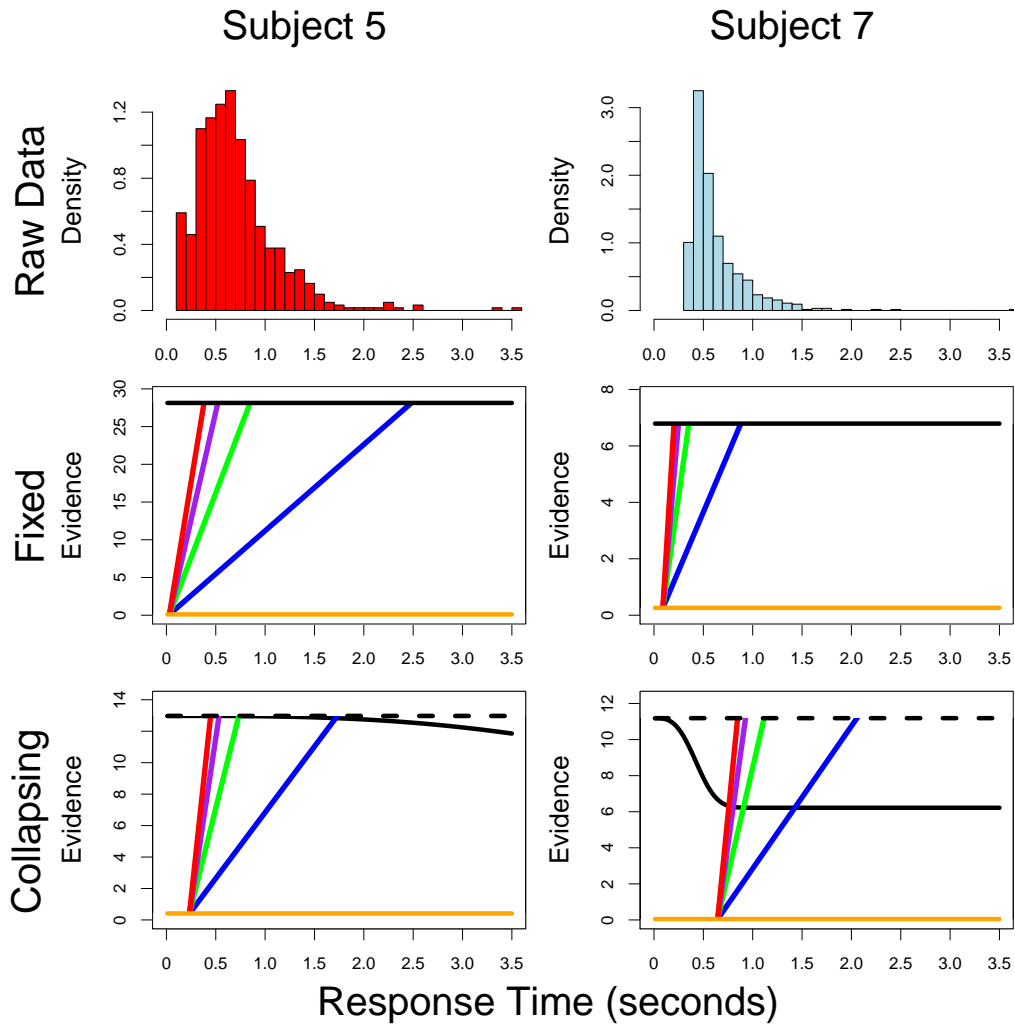


Figure 10: **Inferred Task Representations for Subjects 5 and 7.** The columns correspond to two subjects' data. The top row shows the empirical response time distribution collapsed across choice and stimulus conditions. If the response time distribution is red, the subjects' data was best fit by the time-invariant model (Model 4). If the response time distribution is light blue, the subjects' data was best fit by the time-variant model (Model 16). The middle row shows the task representation inferred by a fixed boundary model, whereas the bottom row shows the task representation inferred by a collapsing boundary model. The drift rates for each coherency condition are shown, color coded according to the key in the middle right panel.



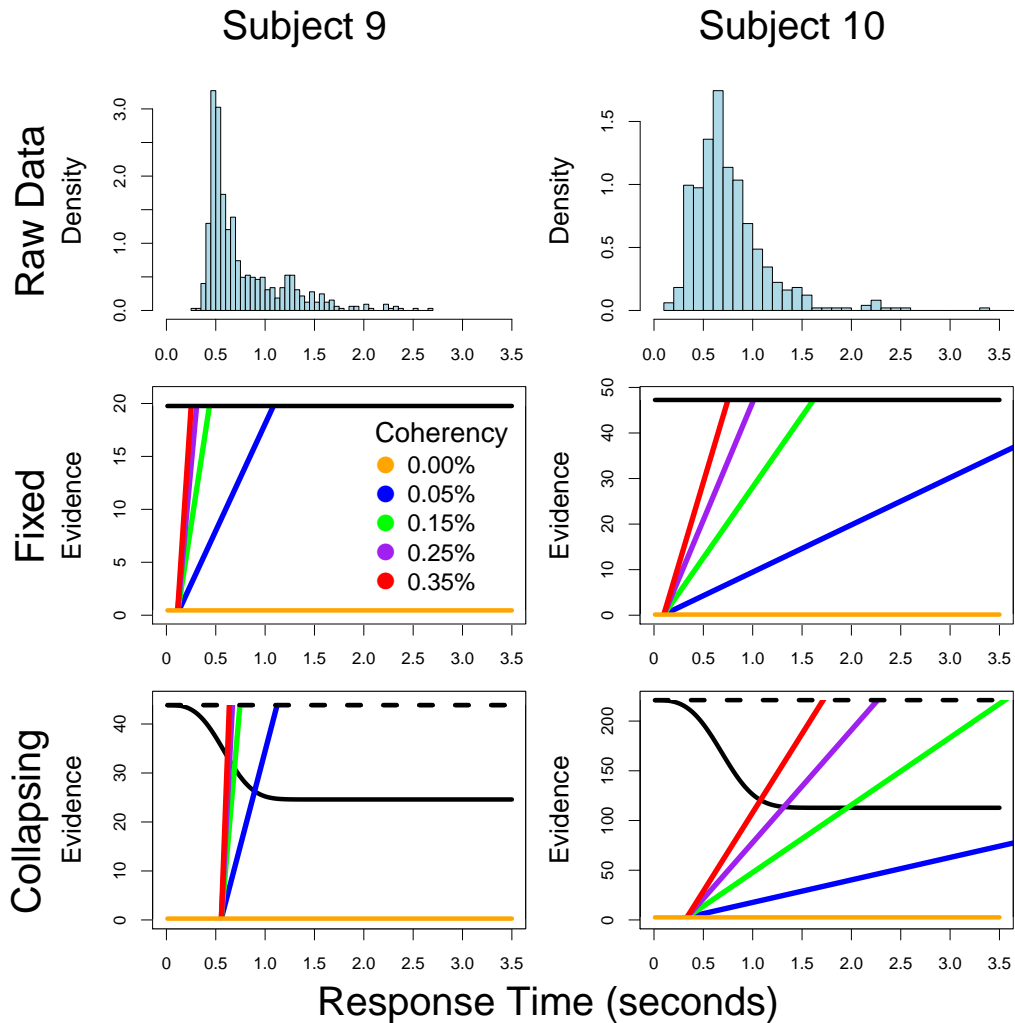


Figure 11: **Inferred Task Representations for Subjects 9 and 10.** The columns correspond to two subjects' data. The top row shows the empirical response time distribution collapsed across choice and stimulus conditions. If the response time distribution is red, the subjects' data was best fit by the time-invariant model (Model 4). If the response time distribution is light blue, the subjects' data was best fit by the time-variant model (Model 16). The middle row shows the task representation inferred by a fixed boundary model, whereas the bottom row shows the task representation inferred by a collapsing boundary model. The drift rates for each coherency condition are shown, color coded according to the key in the middle right panel.

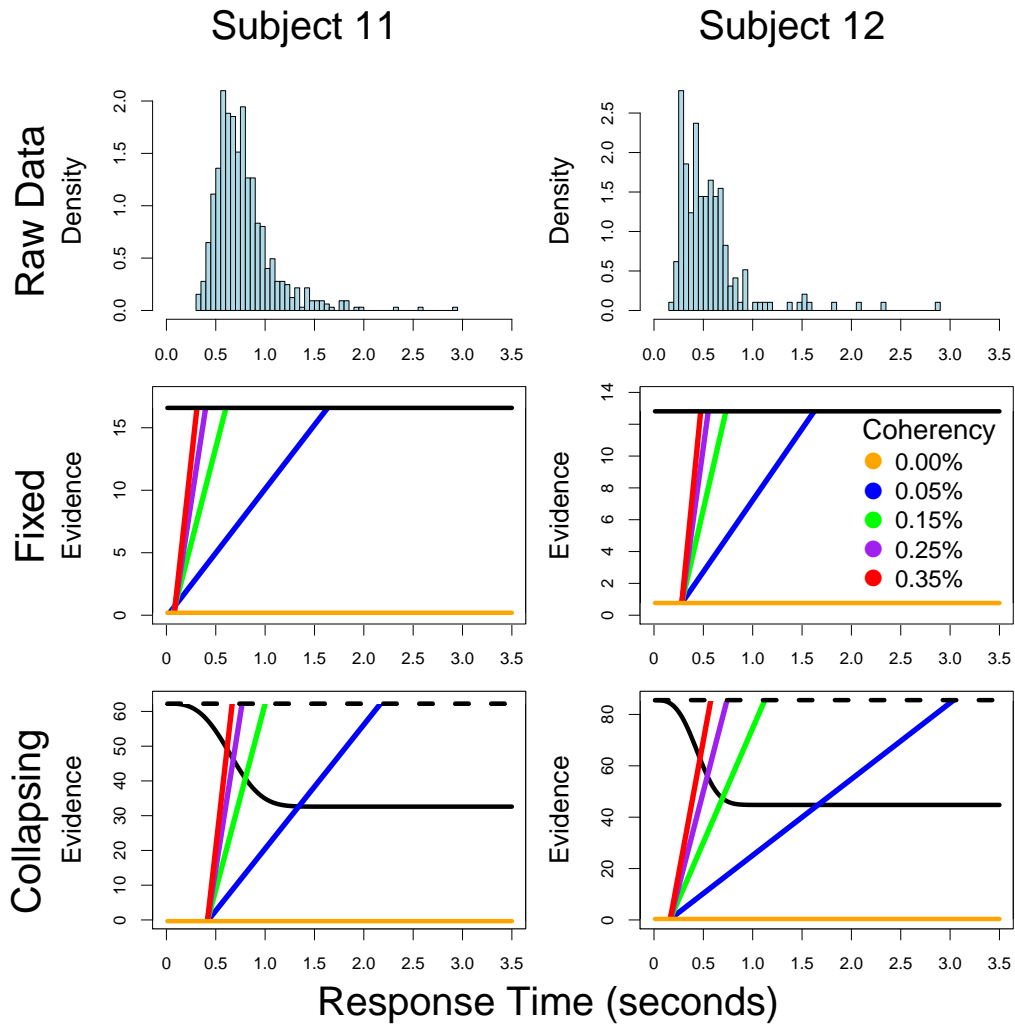


Figure 12: **Inferred Task Representations for Subjects 11 and 12.** The columns correspond to two subjects' data. The top row shows the empirical response time distribution collapsed across choice and stimulus conditions. If the response time distribution is red, the subjects' data was best fit by the time-invariant model (Model 4). If the response time distribution is light blue, the subjects' data was best fit by the time-variant model (Model 16). The middle row shows the task representation inferred by a fixed boundary model, whereas the bottom row shows the task representation inferred by a collapsing boundary model. The drift rates for each coherency condition are shown, color coded according to the key in the middle right panel.

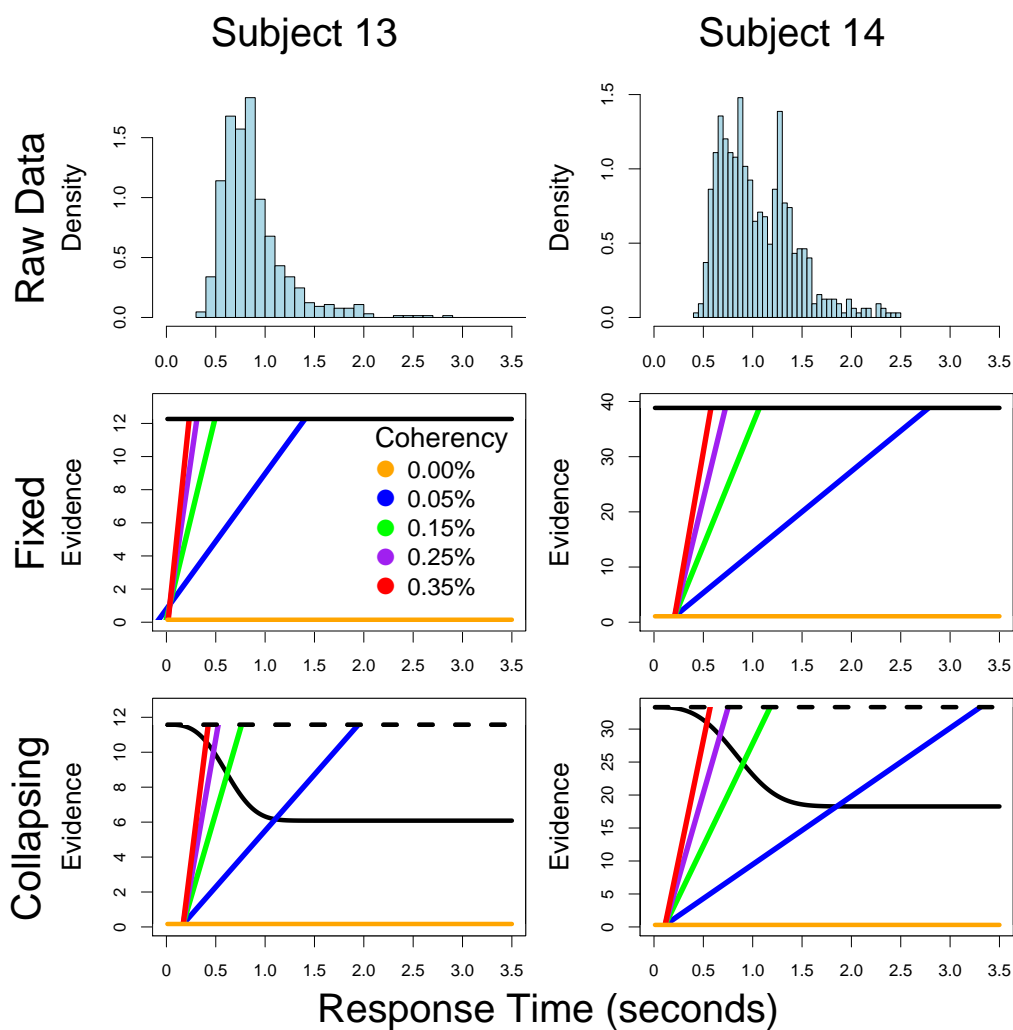


Figure 13: **Inferred Task Representations for Subjects 13 and 14.** The columns correspond to two subjects' data. The top row shows the empirical response time distribution collapsed across choice and stimulus conditions. If the response time distribution is red, the subjects' data was best fit by the time-invariant model (Model 4). If the response time distribution is light blue, the subjects' data was best fit by the time-variant model (Model 16). The middle row shows the task representation inferred by a fixed boundary model, whereas the bottom row shows the task representation inferred by a collapsing boundary model. The drift rates for each coherence condition are shown, color coded according to the key in the middle right panel.

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